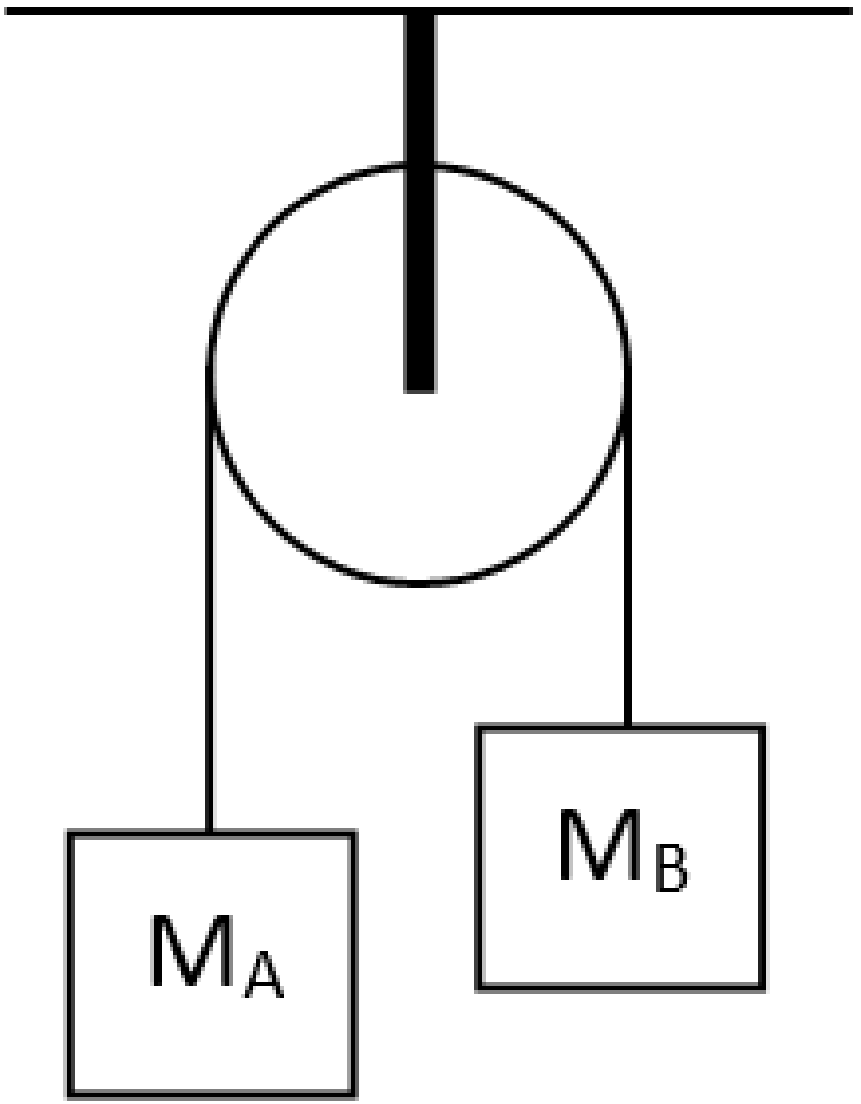
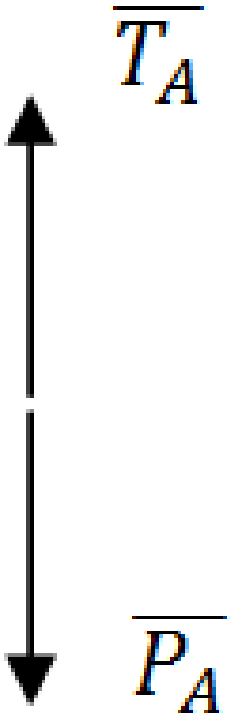
Dinámica

Restricciones. Cuerpos vinculados

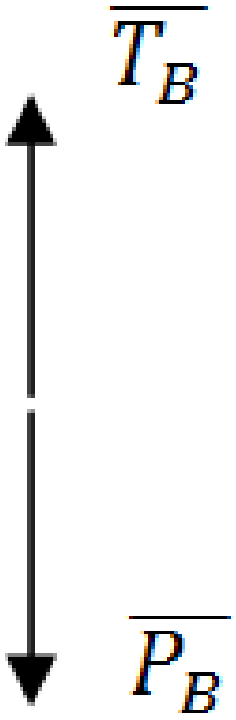
Ejemplo 1 - Datos: M_A, M_B, g. Soga y polea ideal. Calcular las aceleraciones



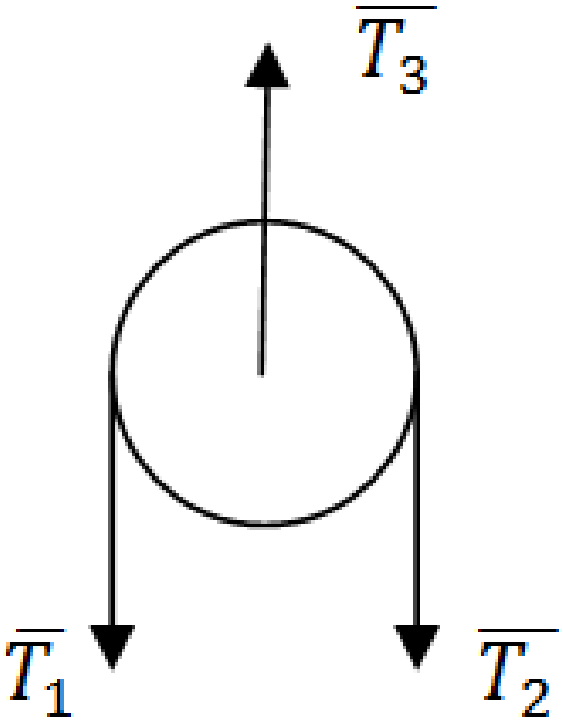
DCL A

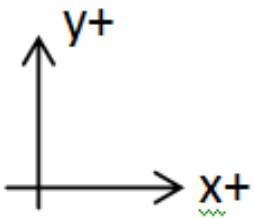


DCL B



DCL Polea





Ecuaciones de movimiento

$$\sum \overline{F_A} = M_A \cdot \overline{a_A}$$

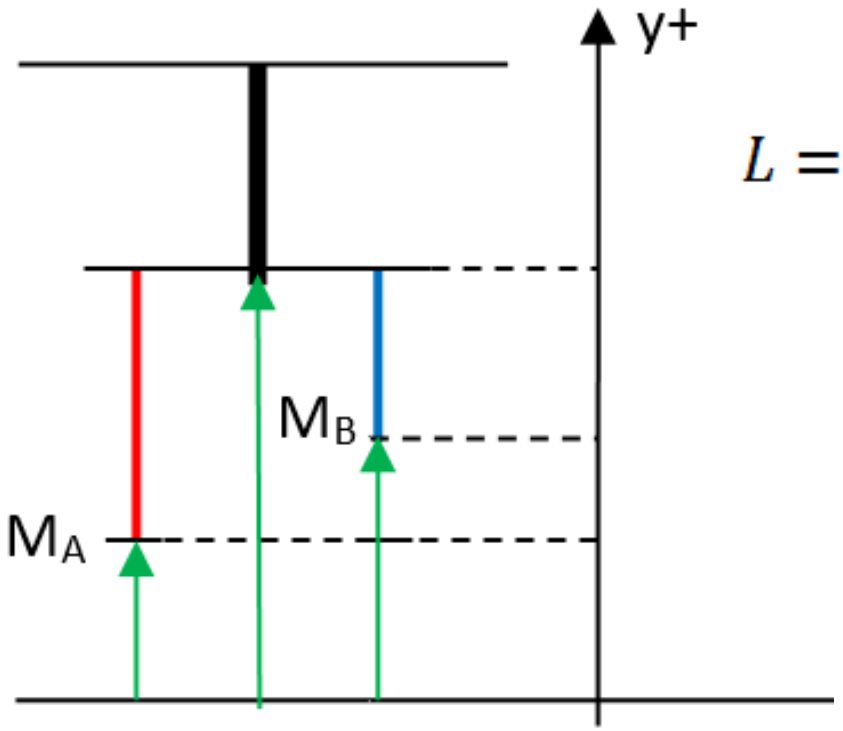
$$y) T_A - P_A = M_A \cdot a_A$$

$$\sum \overline{F_B} = M_B \cdot \overline{a_B}$$

$$y) T_B - P_B = M_B \cdot a_B$$

Vínculo: Soga inextensible

- Longitud constante



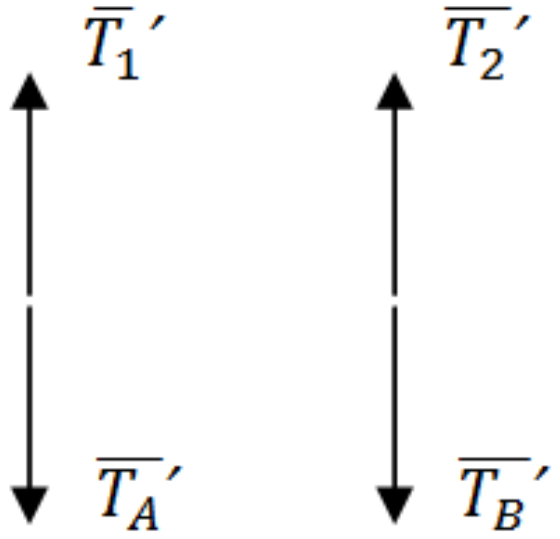
$$L = Y_P - Y_A + Y_P - Y_B$$

$$0 = -a_A - a_B \qquad \frac{d^2}{dt^2}$$

$$a_A = -a_B$$

Vínculo: Soga de masa despreciable

- DCL de cada tramo de soga



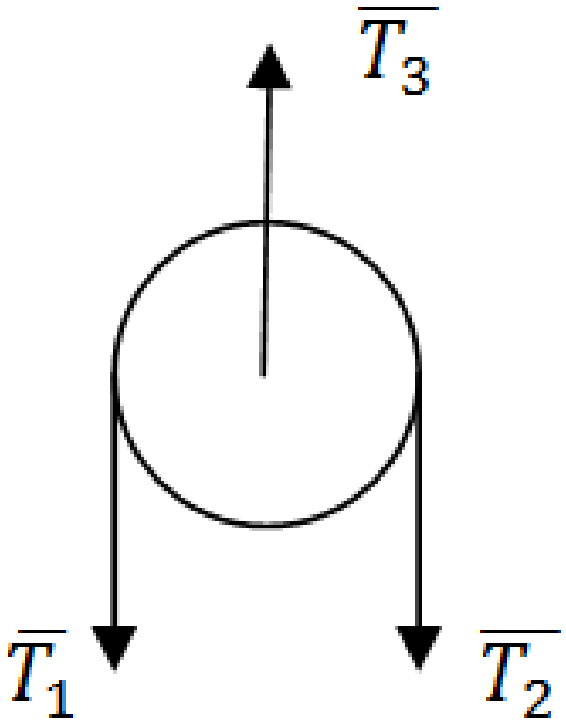
$$\sum \overline{F_S} = M_S \cdot \overline{a}$$

$$T_1 = T_A$$

$$T_2 = T_B$$

Vínculo: polea de masa despreciable

- DCL de la polea (fija)



$$\sum \overline{F_P} = M_P \cdot \overline{a}$$

$$T_1 + T_2 = T_3$$

Rotacion

$$T_1 = T_2$$

- Ecuaciones de movimiento

$$T_A - P_A = M_A \cdot a_A$$

$$T_B - P_B = M_B \cdot a_B$$

- Ecuaciones de vínculo
  - Soga inextensible

$$a_A = -a_B$$

  - Masa despreciable

$$T_1 = T_A = T_2 = T_B = T$$

- Reemplazo los vínculos en las ecuaciones de movimiento

$$T - P_A = M_A \cdot (-a_B)$$

$$T - P_B = M_B \cdot a_B$$

- Resuelvo

$$P_A - P_B = (M_A + M_B) \cdot a_B$$

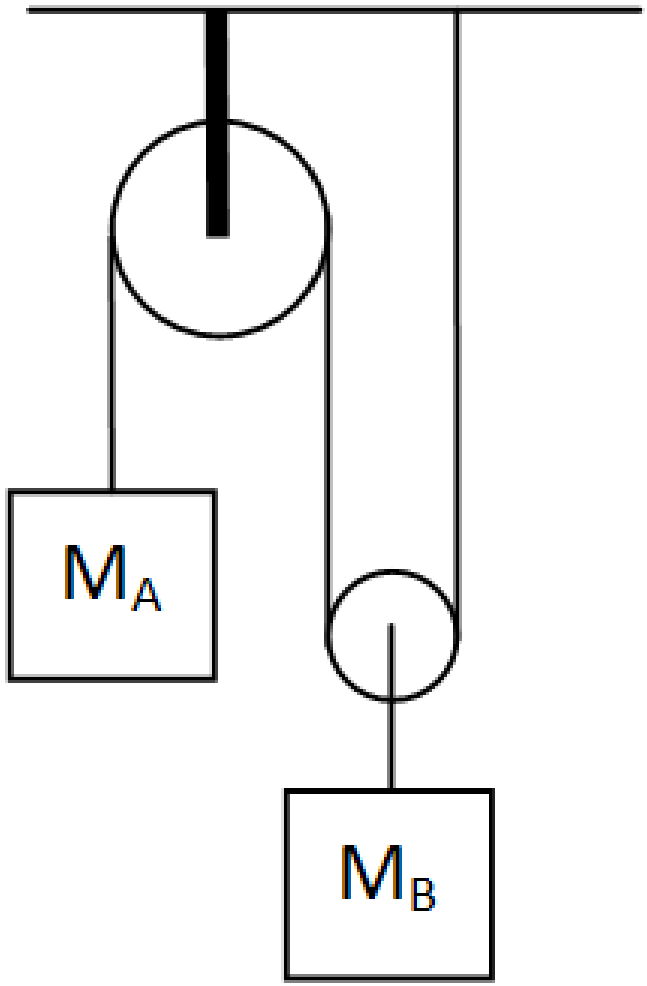
$$a_B = g \cdot \frac{(M_A - M_B)}{(M_A + M_B)}$$

- Respuesta

$$\overrightarrow{a_B} = g \cdot \frac{(M_A - M_B)}{(M_A + M_B)} \hat{j}$$

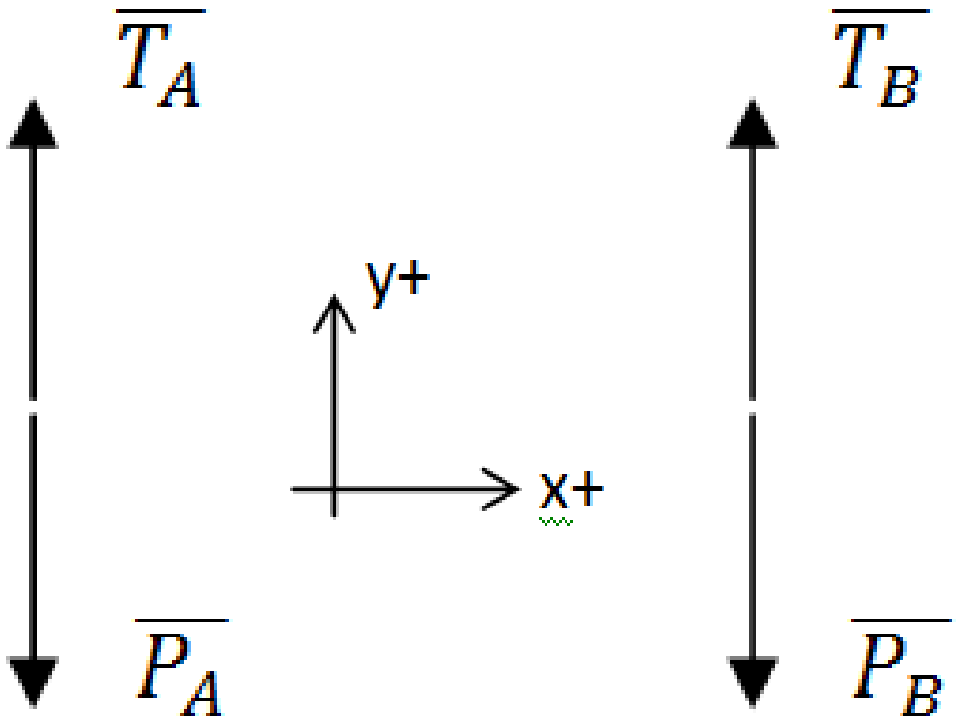
$$\overrightarrow{a_A} = -g \cdot \frac{(M_A - M_B)}{(M_A + M_B)} \hat{j} = g \cdot \frac{(M_B - M_A)}{(M_A + M_B)} \hat{j}$$

Ejemplo 2 - Datos: M_A, M_B, g. Soga y polea ideal. Calcular las aceleraciones



DCL A

DCL B



Ecuaciones de movimiento

$$\sum \overline{F_A} = M_A \cdot \overline{a_A}$$

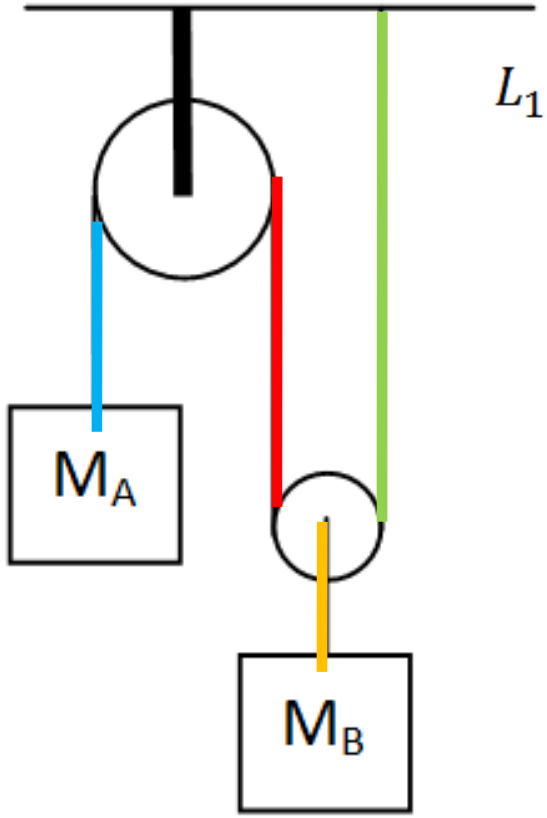
$$y) T_A - P_A = M_A \cdot a_A$$

$$\sum \overline{F_B} = M_B \cdot \overline{a_B}$$

$$y) T_B - P_B = M_B \cdot a_B$$

Vínculos

- Sogas inextensibles (L=cte)



$$L_1 = Y_{PF} - Y_A + Y_{PF} - Y_{PM} + Y_T - Y_{PM}$$

$$0 = -a_A - 2a_{PM}$$

$$a_A = -2a_{PM}$$

$$\frac{d^2}{dt^2}$$

$$L_2 = Y_{PM} - Y_B$$

$$0 = a_{PM} - a_B$$

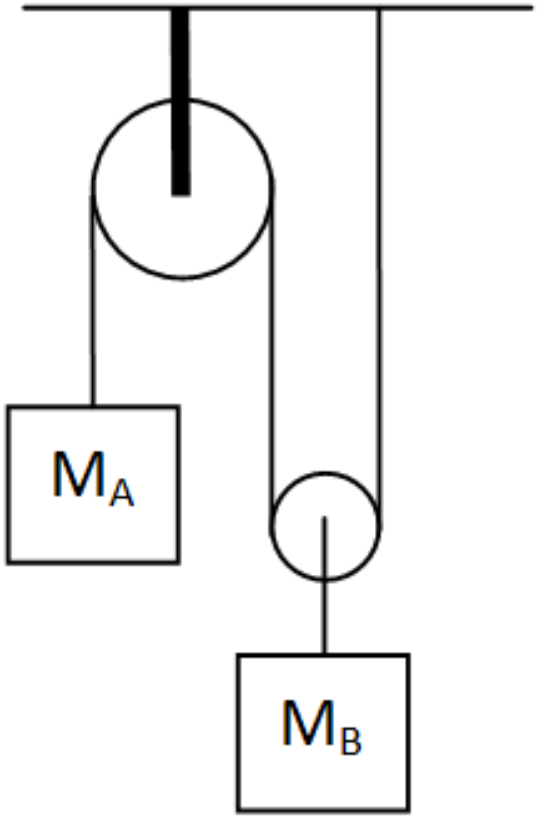
$$a_{PM} = a_B$$

$$\frac{d^2}{dt^2}$$

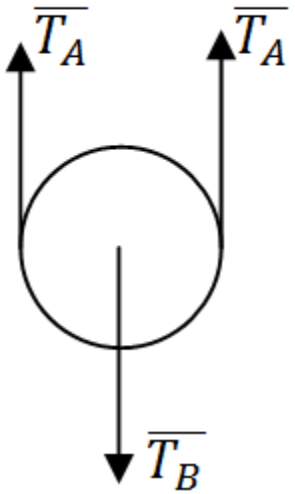
$$a_A = -2a_B$$

Vínculos

- Sogas y polea de masa despreciable



DCL PM



$$\sum \overline{F}_{PM} = M_{PM} \cdot \overline{a_{PM}}$$

$$2T_A = T_B$$

- Ecuaciones de movimiento

$$T_A - P_A = M_A \cdot a_A$$

$$T_B - P_B = M_B \cdot a_B$$

- Ecuaciones de vínculo
  - Soga inextensible

$$a_A = -2a_B$$

  - Masa despreciable

$$2T_A = T_B$$

- Reemplazo los vínculos en las ecuaciones de movimiento

$$T_A - P_A = M_A \cdot (-2a_B)$$

$$2T_A - P_B = M_B \cdot a_B$$

- Resuelvo

$$2P_A - P_B = (4M_A + M_B) \cdot a_B$$

$$a_B = g \cdot \frac{(2M_A - M_B)}{(4M_A + M_B)}$$

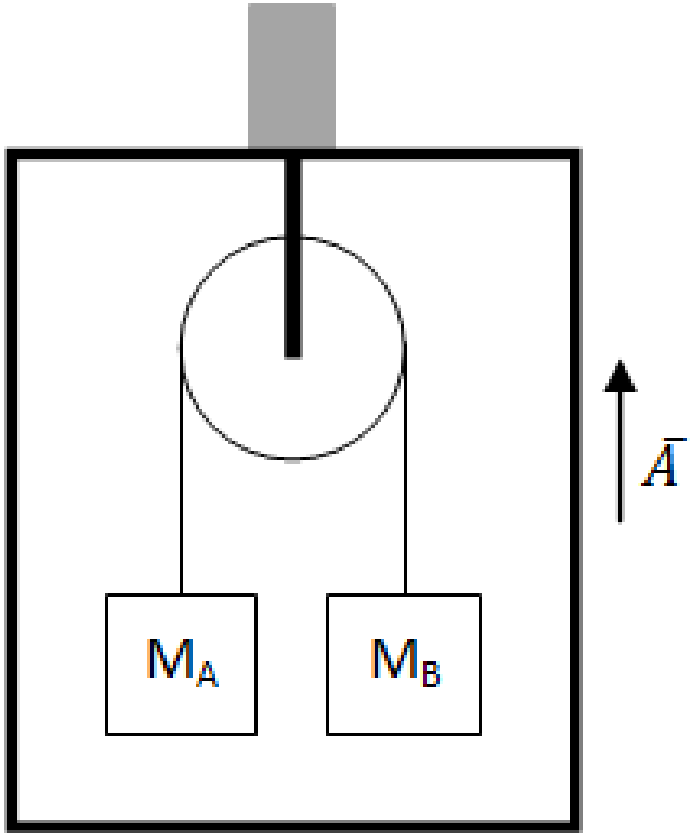
- Respuesta

$$\overline{a_B} = g \cdot \frac{(2M_A - M_B)}{(4M_A + M_B)} \hat{j}$$

$$\overline{a_A} = -2g \cdot \frac{(2M_A - M_B)}{(4M_A + M_B)} \hat{j}$$

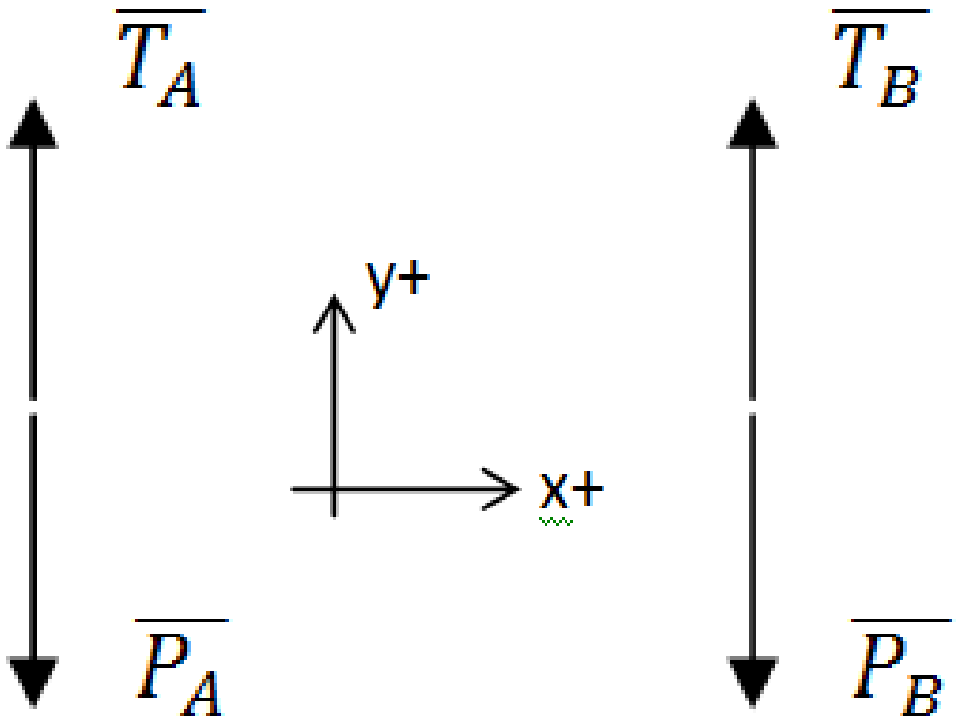
$$\overline{a_A} = 2g \cdot \frac{(M_B - 2M_A)}{(4M_A + M_B)} \hat{j}$$

Ejemplo 1 - Datos: M_A, M_B, g, A. Soga y polea ideal. Calcular las aceleraciones



DCL A

DCL B



Ecuaciones de movimiento

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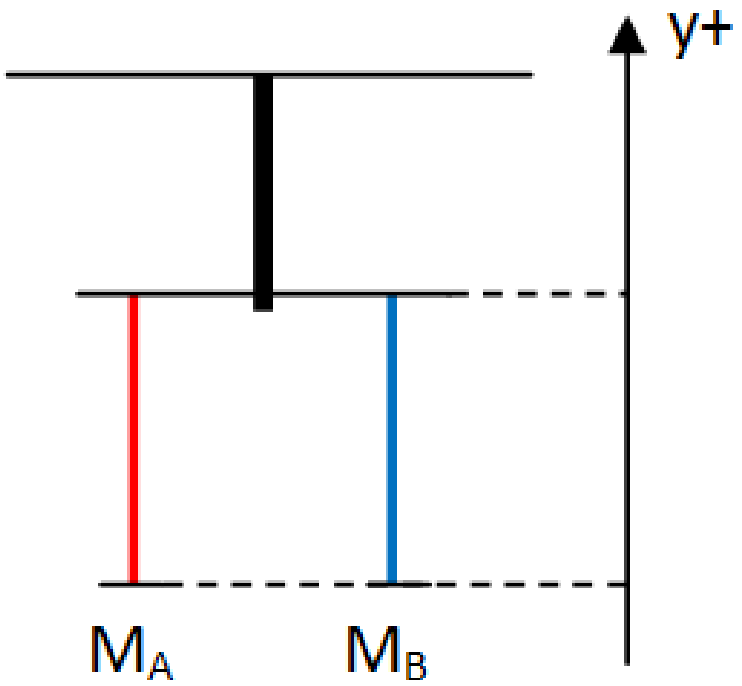
$$y) T_A - P_A = M_A \cdot a_A$$

$$\sum \overline{F_B} = M_B \cdot \overline{a_B}$$

$$y) T_B - P_B = M_B \cdot a_B$$

Vínculos

- Soga inextensible (L=cte)



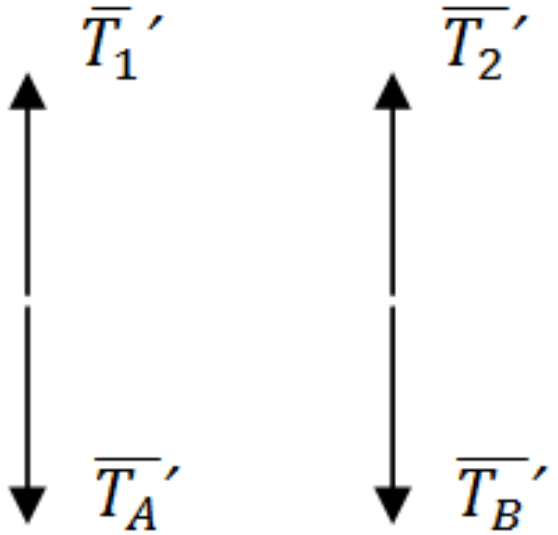
$$L_1 = Y_P - Y_A + Y_P - Y_B$$

$$0 = 2a_P - a_A - a_B \quad \xrightarrow{\frac{d^2}{dt^2}}$$

$$a_A = 2A - a_B$$

Vínculos

- Masa de soga despreciable



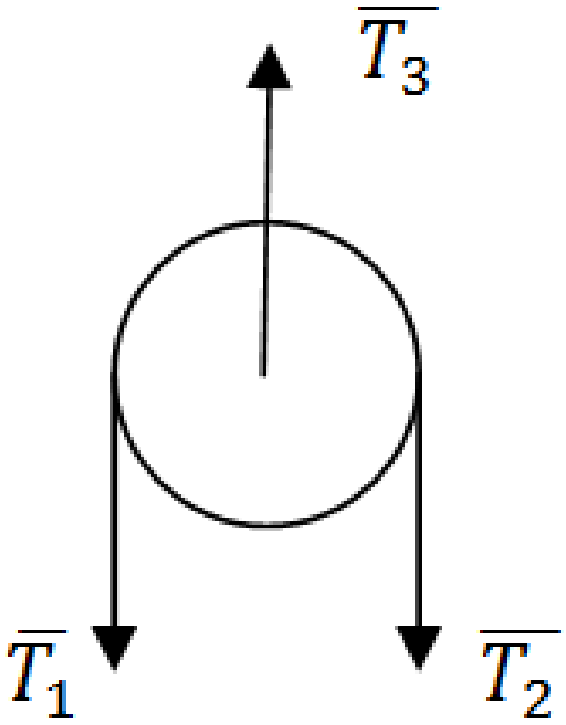
$$\sum \overline{F_S} = M_S \cdot \overline{a}$$

$$T_1 = T_A$$

$$T_2 = T_B$$

Vínculos

- Masa de polea despreciable



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Rotacion

$$T_1 = T_2$$

- Ecuaciones de movimiento

$$T_A - P_A = M_A \cdot a_A$$

$$T_B - P_B = M_B \cdot a_B$$

- Ecuaciones de vínculo
  - Soga inextensible

$$a_A = 2a_P - a_B$$

$$a_A = 2A - a_B$$

  - Masa despreciable

$$T_1 = T_A = T_2 = T_B = T$$

- Reemplazo los vínculos en las ecuaciones de movimiento

$$T - P_A = M_A \cdot (2A - a_B)$$

$$T - P_B = M_B \cdot a_B$$

- Resuelvo

$$P_B - P_A = (M_A + M_B) \cdot a_B - 2M_A A$$

$$a_B = \frac{(M_A - M_B) \cdot g + 2M_A \cdot A}{(M_A + M_B)}$$

- Respuesta

$$\overrightarrow{a_B} = \frac{(M_A - M_B)\cdot g + 2M_A \cdot A}{(M_A + M_B)}\,\hat{j}$$

$$\overrightarrow{a_A} = (2A - a_B)\,\hat{j}$$

$$\overrightarrow{a_A} = \frac{2M_B \cdot A - (M_A - M_B)\cdot g}{(M_A + M_B)}\,\hat{j}$$